Related Rates

For all of these problems, first find exact answers (ie. fractions) and then round to three significant digits.

1. Suppose air is being pumped into a balloon at a rate of 4.5 cubic feet per minute. Find the rate of change of the radius when the radius is 3 feet.

2. A plane is flying on a path to go directly over your home at an altitude of 6 miles. If the distance from the plane to your house is decreasing at a rate of 450 miles per hour when it is 10 miles away, what is the speed of the plane?

3. A camera is watching a shuttle vertically lift off from 2,500 feet away. The displacement function of the ship $s(t) = 50t^2$ is measure in feet while the time t is measured in seconds. Determine the rate of change of the angle of elevation of the camera ten seconds after lift off.

4. Water is being pumped in to a canonical tank at a rate of 6 cubic meters per minute. The tank stands point down and has a height of 10 meters and a base radius of 4 meters.

   (a) At what rate is the water level rising when its level is 5m high?

   (b) As time progresses, what happens to the rate at which the water is rising?

5. Water is being drained out of a canonical tank at a rate of 16 cubic feet per minute. The tank stands point down and has a height of 20 feet and a base radius of 5 feet. How fast is the water level falling when the water is 10 feet deep? The equation for the volume of a cone with radius r and height h is: $V = \frac{1}{3}\pi r^2 h$.

6. Consider an isosceles triangles whose two equal sides have length s and the included angle is θ.

   - Show the area of this triangle is given by the equation: $Area = \frac{1}{2}s^2 \sin\theta$.
   - If θ is changing at a rate of $\frac{1}{4}$ radian per minute, find the rate of change of the area when θ equals $\pi/6$ and $\pi/3$.
   - Explain why the rate of change of the area changes even when the rate of change of θ is constant.

7. Drew is walking towards a street light that is 15 feet tall at a rate of 4 feet per second. If Drew is only 5 feet tall, answer the following questions concerning the moment he is 10 feet from the bas of the street light.

   - What is the rate the tip of Drew's shadow moving?
   - What is the rate the length of his shadow changing? Does this mean it is getting longer or shorter?

8. On a morning when the sun will pass directly over a 40 foot sycamore tree on level ground, it's shadow is 70 feet long at a specific moment in time. At this moment, the angle θ the sun makes with the ground is increasing at a rate of $0.33°/\text{min}$. At what rate is the shadowing decreasing? Express your answer in inches per minute and remember to convert to radians!